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AERODYNAMIC EFFECTS OF METEORITES. A SPECIFIC CASE

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AERODYNAMIC EFFECTS OF METEORITES. A SPECIFIC CASE*

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Meteor bodies begin to experience an intense luminescence and braking at altitudes of the order of 100 kilometers and lower. At these altitudes, the length of the free path of atmosphere molecules is measurable in centimeters and millimeters, and this considerably exceeds the usual dimensions of meteor bodies. That is why discrete molecule collisions with a meteor body may be considered in the study of their motion. At high velocities of meteor body motion (18 to 20 km/sec), every colliding molecule knocks out a considerable number of atoms or molecules from the meteor body crystal lattice and provokes some sort of microexplosions at its surface. At the same time, not only the evaporated mass, but also a simply-fractioned mass consisting of a group of the lattice bound particles is being ejected from the surface of the meteor body. (See refs. 1 and 2.)

The ejection speed of the mass M is lower than the thermal mass (that is, it is lower than the speed corresponding to evaporation temperature), while the quantity of motion (momentum) I, or the corresponding "reaction" impulse of recoil is greater than at ejection of only a gaseous mass, inasmuch as $I \approx \sqrt{ME}$, where E is the ejection energy (ref. 3). A similar process leads to a higher deceleration value or to the increase of the dimensionless coefficient of resistance.

Let us write the conservation of momentum and energy (in a system of coordinates in which air is at rest), as follows:

$$d(Mu) + u_1{}^{\circ}dm + u_2{}^{\circ}dM \tag{1}$$

$$d(Mu^2) + u_1{}^2dm + 2dE_b{}^* = u_2{}^2dM \tag{2}$$

Here M is the mass of the meteor body, m is the mass of molecules of the atmosphere colliding with the meteor body, and u is the meteor body velocity; u_1 and u_2 are the speeds of divergence of

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air molecules and lattice "particles," respectively, from the surface of the meteor body, u_1^o and u_2^o are the velocities of the same, projected normal to the direction of flight, and E_b^* is the intrinsic energy acquired by the meteor body, which partly changes into radiation.

If, at time of impact,

$$\frac{1}{2}\left(\frac{u}{1+\alpha}\right)^2 > \epsilon_k \tag{3}$$

where ϵ_k is the mass density of energies of the meteor body crystal lattice (density of evaporation energy), a vaporization of a certain quantity of lattice particles will take place.

The evaporation will end under the following conditions:

$$\frac{1}{2}\left(\frac{u_k}{1+\alpha}\right)^2 = \epsilon_k \qquad \left(\alpha = \frac{\mu_a}{\mu}\right)$$

where μ is the molecular weight of the lattice, and μ_a is the molecular weight of the atmosphere.

Therefore, the evaporated mass will be determined by the expression,

$$M_1\epsilon_k = \frac{1}{2}m(u^2 - u_k^2) = \frac{1}{2}mu^2 - (1+\alpha)^2 m\epsilon_k$$

Let us write, for simplicity of notation, $dM = M$; $dm = m$. Hence

$$M_i = m\left[\frac{u^2}{2\epsilon_k} - (1+\alpha)^2\right] \tag{4}$$

The evaporated mass will be endowed with an energy:

$$E' = \frac{1}{2}mu_k^2 = (1+\alpha)^2 m\epsilon_k \tag{5}$$

The product "gas" will be endowed with an energy:

$$E_b = \frac{c_v}{\mu} T_i M_i + E'$$

$$= \left[\frac{c_v}{\mu} T_i \frac{u^2}{2\epsilon_k} + (1 + \alpha)^2\left(\epsilon_k - \frac{c_v}{\mu} T_i\right)\right] m$$

$$= m\left[\frac{c_v}{\mu} T_i\left(\frac{u^2}{2\epsilon_k} - (1 + \alpha)^2\right) + (1 + \alpha)^2 \epsilon_k\right] \quad (6)$$

The mean energy density will be:

$$\overline{\epsilon} = \frac{E_b}{m + M_i}$$

$$= \frac{(c_v/\mu) T_i \left((u^2/2\epsilon_k) - (1 + \alpha)^2\right) + (1 + \alpha)^2 \epsilon_k}{1 + u^2/2\epsilon_k - (1 + \alpha)^2} \quad (7)$$

Aside from "evaporation" of the lattice in a certain area, it will be simply breaking up in the area adjacent to the evaporated zone.

Let ϵ^* be the minimum density of energy at which this phenomenon still takes place; then, the total mass M_n of the deformed lattice will be determined by the relation

$$M_n + m = \frac{E_b - \Delta E}{\epsilon^*}$$

provided $\overline{\epsilon} > \epsilon^*$, where ΔE are the losses of energy serving the partial destruction and deformation of the lattice, while the following may be written

$$\Delta E = \eta (M_n - M_i) \epsilon^*$$

where the factor $\eta < 1$ shows what part of energy is irreversibly expended on the deformation of the lattice.

We definitely may write that

$$M_n + m = \left[\frac{E_b}{\epsilon^*} + \eta\left(M_i + m\right)\right]\frac{1}{1 + \eta} \tag{8}$$

From equations (4), (6), and (8), we have

$$M_n = \frac{m}{1 + \eta}\left\{\eta\left(\frac{u^2}{2\epsilon_k} - (1 + \alpha)^2\right) - 1 + \frac{c_v T_i}{\mu\epsilon^*}\left(\frac{u^2}{2\epsilon_k} - (1 + \alpha)^2\right) + (1 + \alpha)^2\frac{\epsilon_k}{\epsilon^*}\right\} \tag{9}$$

with $\eta = 0$ when $\epsilon = \overline{\epsilon}$, and $\eta = \eta_0 < 1$ when $\epsilon^* < \overline{\epsilon}$.

It may be estimated that the difference of velocities $u_1 - u = v_1$, where v_1 is the speed of escape of atmosphere molecules from the meteor body; it is determined by the molecule thermal velocity

$$\frac{v_1^2}{2} = \frac{R}{\mu_a}\frac{T_i^*}{k - 1} = \frac{c_v T_i^*}{\mu_a} \tag{10}$$

where T_i^* is the atmosphere molecule "temperature," not equal to the temperature of evaporation.

The difference of the velocities $u_2 - u = v_2$, where v_2 is the velocity of departure of lattice particles from the surface of the meteor body; it is determined by the relation

$$\frac{v_2^2}{2} = \frac{E_b - \Delta E}{m + M_n} = \epsilon^* \tag{11}$$

The velocity of particle departure, in the case of evaporation only (without taking into account the additional fractioning), will be determined by the relation

$$\begin{aligned}\frac{v_{2i}^2}{2} &= \overline{\epsilon} \\ &= \frac{c_v T_i}{\mu} + \frac{(1 + \alpha)^2\epsilon_k - (c_v/\mu)T_i}{1 + \left(u^2/2\epsilon_k\right) - (1 + \alpha)^2}\end{aligned} \tag{12}$$

Now it is necessary to establish the relationship between the magnitudes u_1^0 and u_2^0 and the magnitudes u_1 and u_2.

For a large number of collisions, the particle departure velocity projected normal to the surface of departure will be:

$$v^*_{1,2} = v^*_{1,2}$$

The velocity $u^\circ_{1,2}$ will be:

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$$u^\circ_{1,2} = u + \kappa v^*_{1,2} = u + \kappa^* \kappa v_{1,2} \tag{13}$$

For an isotropic departure in the hemisphere $\kappa^* = \frac{1}{2}$, the coefficient κ depends on the shape of the body. Since $\kappa = \kappa^* = \frac{1}{2}$ for a spherical body, for a cone with an angle θ at its apex, $\kappa = \sin\theta$; for a plane, $\kappa = 1$, and so forth. Therefore,

$$u^\circ_{1,2} = u + \frac{1}{2}\kappa v_{1,2} \tag{14}$$

Let us take advantage of the expressions

$$dm = S\rho u dt \qquad dM = -dmf(u) \tag{15}$$

where S is the surface of the meteor body cross section,

$$f(u) = \frac{1}{1 + \eta}\left[\left(\eta + \frac{c_v T_i}{\mu \epsilon^*}\right)\left(\frac{u^2}{2\epsilon_k} - (1 + \alpha)^2\right) + \frac{\epsilon_k}{\epsilon^*}(1 + \alpha)^2 - 1\right] \tag{16}$$

Transforming equation (1), we find that

$$-M\frac{du}{dt} = S\rho u^2\left[\frac{u_1{}^\circ}{u} + f(u)\left(\frac{u_2{}^\circ}{u} - 1\right)\right] \tag{17}$$

The usual aerodynamic notation calls for

$$-M\frac{du}{dt} = \frac{c_x}{2}S\rho u^2 \tag{18}$$

Comparing equations (17) and (18), we find that

$$c_x = 2\left[\frac{u_1{}^o}{u} + \left(\frac{u_2{}^o}{u} - 1\right)f(u)\right] = c_x(u) \qquad (19)$$

Transforming equation (19), we may write that

$$c_x = 2\left[1 + \frac{\kappa^*\kappa}{u}\left(v_1 - v_2 f(u)\right)\right]$$

$$= 2 + \frac{\kappa}{u}\left[v_1 + v_2 f(u)\right] \qquad (20)$$

Substituting the values of velocities, we finally obtain

$$c_x = 2 + \frac{x}{u}\left[\sqrt{\frac{RT_1}{(k - 1)\mu_a}} + \frac{\sqrt{2\epsilon^*}}{1 + \eta}\left\{\left(\eta + \frac{RT_1}{(k - 1)\mu\epsilon^*}\right)\left(\frac{u^2}{2\epsilon_k} - (1 + \alpha)^2\right)\right.\right.$$

$$\left.\left. + \frac{\epsilon_k}{\epsilon^*}(1 + \alpha)^2 - 1\right\}\right] \qquad (21)$$

Let us pose

$$k = \frac{5}{3} \qquad \eta = 0 \qquad \mu_a = 30 \qquad \mu = 60 \qquad \alpha = \frac{1}{2}$$

We then shall have

$$c_x = 2 + \frac{1}{2u}\left[\sqrt{\frac{RT_1}{20}} + \sqrt{2\epsilon^*}\left\{\frac{RT_1}{40\epsilon^*}\left(\frac{u^2}{2\epsilon_k} - \frac{9}{4}\right) + \frac{9\epsilon_k}{4\epsilon^*} - 1\right\}\right] \qquad (22)$$

With $u^2 > \epsilon_k > \epsilon^*$, and neglecting the secondary terms, we shall obtain

$$c_x = 2 + \frac{1}{8}\sqrt{\frac{2}{\epsilon^*}}\,\frac{\epsilon_k}{u}\left[9 + \frac{RT_1 u^2}{20\epsilon_k{}^2}\right] \qquad (23)$$

Assuming (for iron)

$$\epsilon^* = \frac{RT_1}{(k - 1)\mu} = \frac{RT_1}{40} = 6 \times 10^9 \frac{\text{erg}}{\text{gr}} \qquad (T_1 = 3{,}000^\circ) \qquad \epsilon_k \approx 7 \times 10^{10} \frac{\text{erg}}{\text{gr}}$$

we shall obtain

$$c_x = 2 + 0.16 \times 10^6\left[\frac{9}{u} + 2.4 \times 10^{-12}u\right] \tag{24}$$

The values c_x calculated after this formula for certain values u expressed in kilometer/sec are as follows:

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u = 10	20	30	40	50	60	70	80	90	100
c = 3.8	3.5	3.6	3.9	4.2	4.5	4.9	5.3	5.6	6.0

With $u = 20$ km/sec, the function $c_x(u)$ has a minimum $c_x = 3.46$. In the general case c_x has a minimum with

$$\frac{u^2}{2\epsilon_k} = -(1 + \alpha)^2 + \frac{(\epsilon_k/\epsilon^*)(1 + \alpha)^2 - 1}{\mu + \dfrac{RT_i}{(k - 1)\mu\epsilon^*}} + \frac{1 + \eta}{\eta + \dfrac{RT_i}{(k - 1)\mu\epsilon^*}} \sqrt{\frac{RT_i}{2(k - 1)\mu_a\epsilon^*}} \tag{25}$$

The value of the shock impulse is

$$\Delta I = -M\Delta u = \Delta m u c_x \qquad (\Delta m = S\rho u\Delta t) \tag{26}$$

On the basis of equation (21), this expression may be written in the form

$$\Delta I = \Delta m\left[2u + a + bu^2\right] \tag{27}$$

where

$$a = \kappa\left\{\sqrt{\frac{RT_i}{(k - 1)\mu_a}} + \frac{\sqrt{2\epsilon^*}}{1 + \eta}\left[\frac{\epsilon_k}{\epsilon^*}(1 + \alpha)^2 - 1 - (1 + \alpha)^2\left(\eta + \frac{RT_i}{(k - 1)\mu\epsilon^*}\right)\right]\right\}$$

$$b = \frac{\kappa\sqrt{2\epsilon^*}}{2\epsilon_k(1 + \eta)}\left(\eta + \frac{RT_i}{(k - 1)\mu\epsilon^*}\right)$$

The magnitude ΔI increases with the increase in velocity. For great velocities

$$\Delta I \approx b\Delta m u^2 \sim \Delta E_0$$

where ΔE_0 is the power of the impact. We thus reach the known conclusion, that in case of an impact with explosion, the quantity of motion is proportional to the energy of the impact (ref. 5).

However, the relationship factors (eq. (27)) differ somewhat from F
a similar relationship for the solid-body impact, inasmuch as a some- 7
what different mechanism of impact takes place in this case than at 0
discrete molecule impact. (The gas produced has a high temperature, and the substance evaporates less, a then becoming smaller.)

It should now be noted that with molecule impact at speeds lower than those provoking "evaporation," $v_2 = 0$ and $c_x = 2u_1^o/u$; furthermore, $u_1^o = \kappa^o u + \kappa^* \kappa v_1$, where for discrete collisions $\kappa^o = 1$, for a compact medium, $\kappa^o < 1$ $(\kappa^o \approx \kappa)$ and it depends on the form of the body and on flowing-around conditions. Thus,

$$c_x = 2\left(\kappa^o + \kappa^* \kappa \frac{v^o}{u}\right) = 2\kappa^o + \kappa\frac{v_1}{u} \approx \left[2 + \frac{v_1}{u}\right]\kappa \tag{28}$$

In case of elastic impact,

$$v_1 = u \quad \text{and} \quad c_x = 4\kappa$$

In case of compact medium collision with high velocities, a flowing-around will take place, and this will reduce the reactive force of departing lattice particles. But the stabilized evaporation regime will also lead to the increase of c_x in comparison with a flowing-around without evaporation. The effective value of the transverse cross section S will then somehow increase, and this leads to the increase of resistance. It may then be estimated that

$$u_1^o = \kappa\left(u + \kappa^{o*} u_1\right) \qquad u_2^o = u + \kappa\kappa^{o*} u_2 \tag{29}$$

where the magnitude $\kappa^{o*} < \kappa^* = \frac{1}{2}$, and it will depend on conditions of flowing-around.

This problem is extremely complex, and it requires a special complementary solution.

It must be particularly noted that at very high velocities, even the force of resistance $F \approx u^3$.

The dependence of c_x on u brought forth, makes sense, as we have shown, even at greater impact velocities than u_k. At smaller impact velocities, it will be almost elastic. It is interesting to note that if $c_x = 4$ in the case of an absolutely elastic impact against a surface, in the case of explosive phenomena c_x has a comparable and even larger value, in spite of the fact that the impact on the surface is not elastic. That is why similar "velocity" collisions are conditionally, somehow, superelastic.

During the analysis of the dependence $c_x = c_x(u)$, it is indicated to take into account that at impact velocities of the order of 10 to 15 km/sec, it is necessary to make use of a more precise general expression (eq. (21)). Then the minimum c_x will be considerably weaker; it will generally disappear, and an almost smooth increase with velocity may be observed.

Let us now take advantage of equation (2). After transformations, we arrive at the following expression:

$$-\frac{dE_b^*}{dM} = uu_2^{\circ} - \frac{1}{2}(u^2 + u_2^2) + \frac{uu_1^{\circ} - 1/2u_1^2}{f(u)}$$

$$= -\left(uv_2(1 - x) + \frac{v_2^2}{2}\right) + \frac{1/2u^2 - \left(uv_1(1 - x) + 1/2v_1^2\right)}{f(u)} \qquad (30)$$

Inasmuch as

$$-\frac{dE_b^*}{dM} = \epsilon_k + \eta\epsilon^*(M_n - M_1) \approx 2\epsilon_k$$

it is possible to determine the values v_1 and T_1^* from equation (30), the value T_1^* being close to the value T_1.

Let us now compute the law of decrease of meteor body mass and velocity as a function of atmosphere pressure.

Let us assume that the meteor body penetrates the atmosphere at an angle φ, counting from the normal. We then have

$$dm = S\rho u dt = S\rho dx = -S\rho \cos\varphi\, dh = \frac{S}{g}\cos\varphi\, dp \qquad (dp = -g\rho dh) \qquad (31)$$

where dx is the element of the path, dh is the altitude variation, a and p is the pressure.

We have

$$M\frac{du}{dt} = -\frac{c_x}{2}S\rho u^2 \qquad dM = -dmf(u) \tag{32}$$

where

$$c_x = 2 + \frac{a}{u} + bu \qquad f(u) = a_0 + b_0u^2$$

$$a_0 = \frac{1}{\sqrt{2\epsilon^*}}\left(\frac{a}{x} - \sqrt{\frac{RT_1}{(k - 1)\mu_a}}\right) \qquad b_0 = \frac{b}{\kappa\sqrt{2\epsilon^*}} \tag{33}$$

By excluding dm, we shall have the equation

$$d\ ln\ M = \frac{du}{u}\frac{f(u)}{1/2c_x}$$

$$= du\frac{a_0 + b_0u^2}{u + 1/2(a + bu^2)} \tag{34}$$

with the initial condition

$$M = M_0 \quad \text{for} \quad u = u_0$$

The solution of this equation is

$$\frac{M}{M_0} = \left(\frac{\frac{1}{2}a + u + \frac{1}{2}bu^2}{\frac{1}{2}a + u_0 + \frac{1}{2}bu_0{}^2}\right)^{2b_0/b^2} \exp \Psi(u,u_0) \tag{35}$$

$$\Psi(u,u_0) = \frac{2\sqrt{a_0} - 2b_0(2 - ab)/b^2}{\sqrt{ab - 1}}\left(\text{arc tan}\frac{bu + 1}{\sqrt{ab - 1}} - \text{arc tan}\frac{bu_0 + 1}{\sqrt{ab - 1}}\right)$$

$$+ \frac{2b_0}{b}(u - u_0)$$

We further have

$$\frac{\cos\varphi\ dp}{g} = -\frac{dM}{sf(u)} = -\frac{dM}{s(a_0 + b_0 u)} \tag{36}$$

Inasmuch as, for a body of any shape

$$\frac{dM}{S} = A\delta^{2/3} dM^{1/3}$$

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where A is the dimensionless parameter depending upon the shape of the body and its revolving in motion (for a uniformly rotating sphere)

$$A = \frac{3}{\pi}\left(\frac{4\pi}{3}\right)^{2/3}$$

equation (35) has the form

$$\frac{\cos\varphi\ dp}{g} = -\frac{dM^{1/3}}{(a_0 + b_0 u)} A\delta^{2/3} \qquad M = M_0 \quad \text{for} \quad p = 0 \tag{37}$$

Taking into account equation (34), we have

$$\frac{\cos\varphi}{gA\delta^{2/3}} p = \int_u^{u_0} \frac{dM^{1/3}}{a_0 + b_0 u} = \Phi(u) \tag{38}$$

Knowing the velocities for the given altitudes, it is possible to find $p = p(h)$, and then $M = M(h)$.

In conclusion, let us point out that for velocity impacts, the magnitude c_x is greater than the general theory predicts. That is why, for the given deceleration, the density of the atmosphere must be several times lower than was earlier calculated by means of meteor data.

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